

HEP-TH/9703030

CERN-TH/97-34

THU-97/06

UTFA-97/06

MATRIX STRING THEORY

ROBBERT DIJKGRAAF

*Department of Mathematics
University of Amsterdam, 1018 TV Amsterdam*

ERIK VERLINDE

TH-Division, CERN, CH-1211 Geneva 23

and

*Institute for Theoretical Physics
University of Utrecht, 3508 TA Utrecht*

and

HERMAN VERLINDE

*Institute for Theoretical Physics
University of Amsterdam, 1018 XE Amsterdam*

ABSTRACT

Via compactification on a circle, the matrix model of M-theory proposed by Banks et al suggests a concrete identification between the large N limit of two-dimensional $\mathcal{N} = 8$ supersymmetric Yang-Mills theory and type IIA string theory. In this paper we collect evidence that supports this identification. We explicitly identify the perturbative string states and their interactions, and describe the appearance of D-particle and D-membrane states.

By definition, M-theory is the 11-dimensional theory that via compactification on a circle S^1 is equivalent to ten dimensional type IIA string theory [1, 2]. The string coupling constant g_s emerges in this correspondence as the radius of the S^1 , while the particles with non-zero KK-momentum along the S^1 are identified with the D-particles of the IIA model. According to the matrix theory proposal put forward in [3], the full dynamics of M-theory can be captured by means of an appropriate large N limit of supersymmetric matrix quantum mechanics.

In the original correspondence with the type IIA string, the matrix degrees of freedom find their origin in the collective dynamics of the D-particles [4, 5, 6]. The key new ingredient in the approach of [3], however, is that the large N limit effectively accomplishes a decompactification of the extra 11th direction, which therefore should be treated on the exact same footing as all the other uncompactified dimensions. This insight, albeit still conjectural, provides a number of important new theoretical tools. Namely, by interchanging the original role of the 11-th direction with that of one of the other directions, one in principle achieves a concrete identification of the complete non-perturbative particle spectrum of string theory in a given dimension with a relatively convenient subset of states (namely all states that can be made up from infinitely many D-particles) of string theory compactified to one dimension less. In this way, much of the recently developed D-brane technology is upgraded by one dimension and has become directly applicable in the study of M-theory compactifications.

In this paper we aim to elaborate this new viewpoint for the simplest compactification of M-theory, namely on S^1 . Following the current approach in matrix theory [3, 7], the compactification on this S^1 is achieved by reinterpreting the infinite dimensional matrices X^i as covariant derivatives D_i (written in a Fourier mode basis) of a large N gauge field defined on the S^1 . In the original D-particle language, this procedure in fact amounts to applying a T-duality transformation along the S^1 -directions, thereby turning the D-particles into D-strings. Adopting this approach, we have cast matrix theory into the form of a two-dimensional $\mathcal{N} = 8$ supersymmetric $U(N)$ Yang-Mills theory with the Lagrangian*

$$S = \frac{1}{2\pi} \int \text{tr} \left((D_\mu X^i)^2 + \theta^T \not{D} \theta + g_s^2 F_{\mu\nu}^2 - \frac{1}{g_s^2} [X^i, X^j]^2 + \frac{1}{g_s} \theta^T \gamma_i [X^i, \theta] \right). \quad (1)$$

Here the 8 scalar fields X^i are $N \times N$ hermitian matrices, as are the 8 fermionic fields θ_L^α and $\theta_R^{\dot{\alpha}}$. The fields X^i , θ^α , $\theta^{\dot{\alpha}}$ transform respectively in the $\mathbf{8}_v$ vector, and $\mathbf{8}_s$ and $\mathbf{8}_c$ spinor representations of the $SO(8)$ R-symmetry group of transversal rotations. The two-dimensional world-sheet is taken to be a cylinder parametrized by coordinates (σ, τ) with σ between 0 and 2π . The fermions are taken in the Ramond sector, and there is no

*Here we work in string units $\alpha' = 1$. A derivation of (1) from matrix theory and a discussion of our normalizations is given in the appendix.

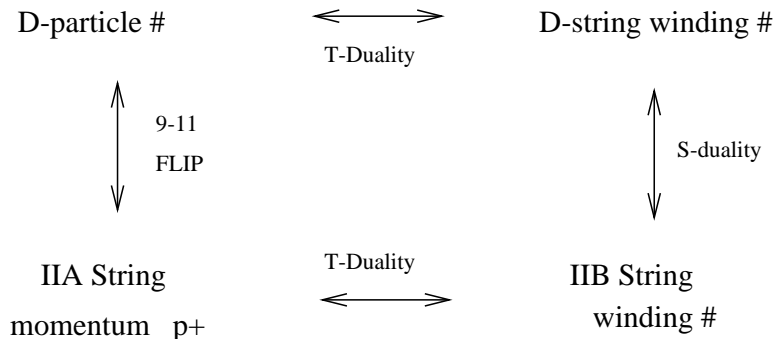


Fig. 1: *The duality diagram that relates the matrix string theory to large N D-particle quantum mechanics and the interpretation of N in the models.*

projection on particular fermion number. According to the matrix model philosophy, we will consider this theory in the limit $N \rightarrow \infty$.

Note that the same set of fields feature in the Green-Schwarz light-cone formalism of the type II superstring, except that here they describe non-commuting matrices [3]. Indeed, the eigenvalues of the above matrix coordinates X^i are from our point of view identified with the coordinates of the fundamental type IIA string, since, relative to the original starting point of large N D-particle quantum mechanics on $S^1 \times R^8$, we have applied an 11-9 flip that interchanges the role of the 11-th and 9-th directions of M-theory. The interpretation of N in this matrix string theory is as (proportional to) the total light-cone momentum p_+ , see *fig. 1*.

In the following sections we will elaborate this correspondence in some detail. We will begin with identifying the complete perturbative string spectrum, which arises in the IR limit of the two-dimensional SYM theory. This correspondence is based on the exact equivalence between a (free) second-quantized string spectrum and the spectrum of a two-dimensional S_N orbifold sigma-model[†]. We will then proceed to study the string interactions from the SYM point of view. Finally, we will describe how D-particles and other D-branes naturally appear in this theory.

While this paper was being written, the preprint [10] appeared, in which closely related results are reported. We also became aware of the earlier work [11], where the formulation of nonperturbative string theory by means of two-dimensional SYM theory was first proposed and where some of our results were independently obtained. This approach was also anticipated in [12, 13]. Similar points of view relating strings and matrices are advocated in, among others, [14, 15].

[†]This new representation of second quantized string theory was first pointed out in [8] and elaborated in more detail in [9].

In the interpretation of the $\mathcal{N} = 8$ SYM model as a matrix string, the usual YM gauge coupling (which has dimension 1/length in two dimensions) is given in terms of the string coupling as $g_{YM}^{-2} = \alpha' g_s^2$. The dependence on the string coupling constant g_s can be absorbed in the area dependence of the two-dimensional SYM model. In this way g_s scales inversely with world-sheet length. The free string at $g_s = 0$ is recovered in the IR limit. In this IR limit, the two-dimensional gauge theory model is strongly coupled and we expect a nontrivial conformal field theory to describe the IR fixed point.

Rather standard argumentation determines that the conformal field theory that describes this IR limit is the $\mathcal{N} = 8$ supersymmetric sigma model on the orbifold target space

$$S^N \mathbf{R}^8 = (\mathbf{R}^8)^N / S_N, \quad (2)$$

see e.g. [16]. First we observe that in the $g_s = 0$ limit the fields X and θ will commute. This means that we can write the matrix coordinates as

$$X^i = U x^i U^{-1} \quad (3)$$

with $U \in U(N)$ and x^i a diagonal matrix with eigenvalues x_1^i, \dots, x_N^i . That is, x^i takes values in the Cartan subalgebra of $U(N)$. This leads to a description of the model in terms of N Green-Schwarz light-cone coordinates $x_I^i, \theta_I^\alpha, \theta_I^{\dot{\alpha}}$ with $I = 1, \dots, N$.

The complete correspondence with a free second-quantized string Hilbert space in the $N \rightarrow \infty$ limit involves the twisted sectors. The only gauge invariant quantity is the set of eigenvalues of the matrices X^i . Therefore, if we go around the space-like S^1 of the world-sheet, the eigenvalues can be interchanged and the fields $x_I^i(\sigma)$ can be multivalued. Specifically, we have to allow for configurations with

$$x^i(\sigma + 2\pi) = g x^i(\sigma) g^{-1}, \quad (4)$$

where the group element g takes value in the Weyl group of $U(N)$, the symmetric group S_N . The eigenvalue fields $x_I^i(\sigma)$ thus take values on the orbifold space $S^N \mathbf{R}^8$. These twisted sectors are depicted in *fig. 2* and correspond to configurations with strings with various lengths.

The Hilbert space of this S_N orbifold field theory is decomposed into twisted sectors labeled by the conjugacy classes of the orbifold group S_N [9],

$$\mathcal{H}(S^N \mathbf{R}^8) = \bigoplus_{\text{partitions } \{N_n\}} \mathcal{H}_{\{N_n\}}. \quad (5)$$

Here we used that for the symmetric group, the conjugacy classes $[g]$ are characterized by partitions $\{N_n\}$ of N

$$\sum_n n N_n = N, \quad (6)$$

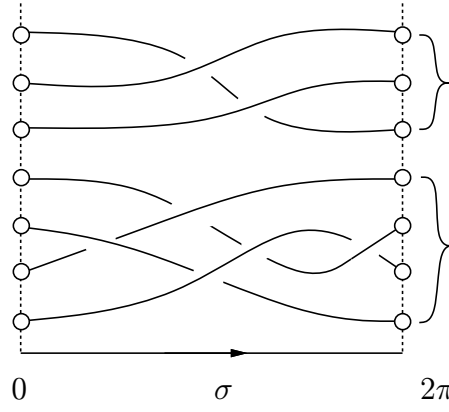


Fig. 2: A twisted sector corresponds to a configuration with strings of various lengths.

where N_n denotes the multiplicity of the cyclic permutation (n) of n elements in the decomposition of g

$$[g] = (1)^{N_1} (2)^{N_2} \dots (s)^{N_s}. \quad (7)$$

In each twisted sector, one must further keep only the states invariant under the centralizer subgroup C_g of g , which takes the form

$$C_g = \prod_{n=1}^s S_{N_n} \times \mathbf{Z}_n^{N_n} \quad (8)$$

where each factor S_{N_n} permutes the N_n cycles (n) , while each \mathbf{Z}_n acts within one particular cycle (n) .

Corresponding to this factorisation of $[g]$, we can decompose each twisted sector into the product over the subfactors (n) of N_n -fold symmetric tensor products of appropriate smaller Hilbert spaces $\mathcal{H}_{(n)}$

$$\mathcal{H}_{\{N_n\}} = \bigotimes_{n>0} S^{N_n} \mathcal{H}_{(n)} \quad (9)$$

where[‡]

$$S^N \mathcal{H} = \left(\underbrace{\mathcal{H} \otimes \dots \otimes \mathcal{H}}_{N \text{ times}} \right)^{S_N}. \quad (10)$$

The spaces $\mathcal{H}_{(n)}$ in (9) denote the \mathbf{Z}_n invariant subsector of the space of states of a single string on $\mathbf{R}^8 \times S^1$ with winding number n . We can represent this space via a sigma model

[‡]Here the symmetrization is assumed to be compatible with the grading of \mathcal{H} . In particular for pure odd states S^N corresponds to the exterior product \bigwedge^N .

of n coordinate fields $x_I^i(\sigma) \in X$ with the cyclic boundary condition

$$x_I^i(\sigma + 2\pi) = x_{I+1}^i(\sigma), \quad I \in (1, \dots, n). \quad (11)$$

We can glue the n coordinate fields $x_I(\sigma)$ together into one single field $x(\sigma)$ defined on the interval $0 \leq \sigma \leq 2\pi n$. Hence, relative to the string with winding number one, the oscillators of the long string that generate $\mathcal{H}_{(n)}$ have a fractional $\frac{1}{n}$ moding. The group \mathbf{Z}_n is generated by the cyclic permutation

$$\omega : x_I \rightarrow x_{I+1} \quad (12)$$

which via (11) corresponds to a translation $\sigma \rightarrow \sigma + 2\pi$. Thus the \mathbf{Z}_n -invariant subspace consists of those states for which the fractional left-moving minus right-moving oscillator numbers combined add up to an integer.

It is instructive to describe the implications of this structure for the Virasoro generators $L_0^{(i)}$ of the individual strings. The total L_0 operator of the S_N orbifold CFT in a twisted sector given by cyclic permutations of length n_i decomposes as

$$L_0^{tot} = \sum_i \frac{L_0^{(i)}}{n_i}. \quad (13)$$

Here $L_0^{(i)}$ is the usual canonically normalized operator in terms of the single string coordinates $x(\sigma), \theta(\sigma)$ defined above. The meaning of the above described \mathbf{Z}_n projections is that it requires that the contribution from a single string sector to the total world-sheet translation generator $L_0^{tot} - \bar{L}_0^{tot}$ is integer-valued. From the individual string perspective, this means that $L_0^{(i)} - \bar{L}_0^{(i)}$ is a multiple of n_i .

To recover the Fock space of the second-quantized type IIA string we now consider the following large N limit. We send $N \rightarrow \infty$ and consider twisted sectors that typically consist of a finite number of cycles, with individual lengths n_i that also tend to infinity in proportion with N . The finite ratio n_i/N then represents the fraction of the total p_+ momentum (which we will normalize to $p_+^{tot} = 1$) carried by the corresponding string[§]

$$p_+^{(i)} = \frac{n_i}{N}. \quad (14)$$

So, in the above terminology, only long strings survive. The usual oscillation states of these strings are generated in the orbifold CFT by creation modes $\alpha_{-k/N}$ with k finite. Therefore, in the large N limit only the very low-energy IR excitations of the S_N -orbifold CFT correspond to string states at finite mass levels.

[§]The fact that the length n_i of the individual strings specifies its light-cone momentum is familiar from the usual light-cone formulation of string theory [17].

The \mathbf{Z}_n projection discussed above in this limit effectively amounts to the usual uncompactified level-matching conditions $L_0^{(i)} - \bar{L}_0^{(i)} = 0$ for the *individual* strings, since all single string states for which $L_0 - \bar{L}_0 \neq 0$ become infinitely massive at large N . The total mass-shell condition reads (here we put the string tension equal to $\alpha' = 1$)

$$p_-^{tot} = NL_0^{tot} \quad (15)$$

and we recover the mass-shell conditions of the individual strings by decomposing L_0^{tot} as in (13), via the definition of the individual p_- light-cone momentum as

$$p_-^{(i)} = \frac{NL_0^{(i)}}{n_i} \quad (16)$$

which combined with (14) gives the usual relation $p_+^{(i)} p_-^{(i)} = L_0^{(i)}$. All strings therefore indeed have the same string tension.

INTERACTIONS

We have seen that the super-Yang-Mills model in the IR limit gives the free light-cone quantization of the type IIA string in terms of an orbifold sigma model, that describes the completely broken phase of the $U(N)$ gauge symmetry. The twisted sectors of the orbifold describe multi-string states and are superselection sectors in the non-interacting model — the number of strings is conserved for $g_s = 0$.

The conservation of string number and of individual string momenta will be violated if we turn on the interactions of the 1+1-dimensional SYM theory. The idea is that by relaxing the strict IR limit, one gradually needs to include configurations in which the non-abelian symmetry gets restored in some small space-time region. Indeed, if at some point in the (σ, τ) plane two eigenvalues x_I and x_J coincide, we enter a phase where an unbroken $U(2)$ symmetry is restored, and we should thus expect that for non-zero g_s , there will be a non-zero transition amplitude between states that are related by a simple transposition of these two eigenvalues[¶]. In the IR S_N orbifold theory, such a process will correspond to a local interaction on the two-dimensional world-sheet, which can be seen to correspond to the elementary joining and splitting of strings.

To see this more explicitly, consider a configuration that connects two different sectors, labeled by S_N group elements that are related by multiplication by a simple transposition. It is easy to see that this simple transposition connects a state with, say, one string represented by a cycle (n) decays into a state with two strings represented by a permutation that is a product of two cycles $(n_1)(n_2)$ with $n_1 + n_2 = n$, or vice versa. So the numbers of incoming and outgoing strings differ by one. Pictorially, what takes place is that the two coinciding eigenvalues connect or disconnect at the intersection point, and as illustrated in *fig. 3*. this indeed represents an elementary string interaction.

[¶]This observation was first made in [11].

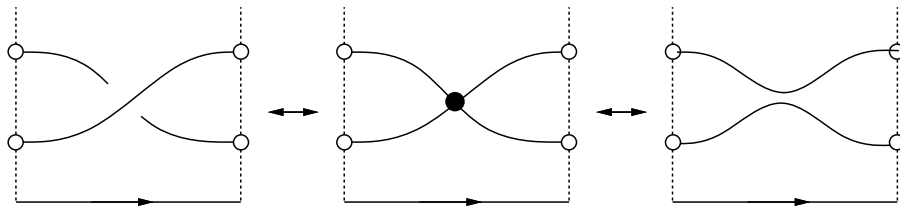


Fig. 3: *The splitting and joining of strings occurs if two eigenvalues coincide.*

In the CFT this interaction is represented by a local operator, and thus according to this physical picture one may view the SYM theory as obtained via a perturbation of the S_N -orbifold conformal field theory. In first order, this perturbation is described via a modification of the CFT action

$$S = S_{CFT} + \lambda \int d^2z V_{int} \quad (17)$$

where V_{int} is an appropriate (e.g. space-time supersymmetric) twist operator, that generates the just described simple transposition of eigenvalues. We now claim that this joining and splitting process is indeed first order in the coupling constant g_s as defined in the SYM Lagrangian (1). Instead of deriving this directly in the strongly coupled SYM theory, we analyze the effective operator that produces such an interaction in the IR conformal field theory. The identification of λ in (17) in terms of the coupling constants in (1) via

$$\lambda \sim g_s \sqrt{\alpha'} \quad (18)$$

requires that this local interaction vertex V_{int} must have scale dimension $(\frac{3}{2}, \frac{3}{2})$ under scale transformations on the two-dimensional world-sheet. We will now verify that this is indeed the case.

THE INTERACTION VERTEX

It is clear from the above discussion that the interaction vertex V_{int} will be a twist field that interchanges two eigenvalues, say x_1 and x_2 . It acts therefore as a \mathbf{Z}_2 reflection on the relative coordinate $x_1 - x_2$. This coordinate has 8 components, so the total conformal dimension of the corresponding superconformal field (including the fermionic contribution) is $(1, 1)$. The corresponding descendent operator that one can include in the action has dimensions $(\frac{3}{2}, \frac{3}{2})$. Note that this is an irrelevant operator, that disappears in the IR limit. In fact, the corresponding coupling constant has dimension -1 . Since the world-sheet length-scale is inversely proportional to the string coupling g_s , we immediately see

that this interaction linear in g_s , which is what we set out to establish. In fact, we will show that the twist field is uniquely characterized as the least irrelevant operator in the CFT that is both space-time supersymmetric and Lorentz invariant.

Let us do this computation in a bit more detail. First we recall that in the CFT description of the second-quantized GS light-cone string the spacetime supercharges are given by (in the normalization $p_+^{tot} = 1$)

$$Q^\alpha = \frac{1}{\sqrt{N}} \oint d\sigma \sum_{I=1}^N \theta_I^\alpha, \quad Q^{\dot{\alpha}} = \sqrt{N} \oint d\sigma G^{\dot{\alpha}}, \quad (19)$$

with

$$G^{\dot{\alpha}}(z) = \sum_{I=1}^N \gamma_{\alpha\dot{\alpha}}^i \theta_I^a \partial x_I^i \quad (20)$$

the generators of the total $\mathcal{N} = 8$ world-sheet supersymmetry algebra, with similarly defined right-moving charges.

Concentrating on the left-moving sector, let us analyze the twist fields of the \mathbf{Z}_2 twist that interchanges two eigenvalues, say x_1 with x_2 and similarly θ_1 and θ_2 . One first goes over to the eigenvectors $x_\pm = x_1 \pm x_2$, $\theta_\pm = \theta_1 \pm \theta_2$. The \mathbf{Z}_2 acts on the minus components as

$$x_-^i, \theta_-^\alpha \rightarrow -x_-^i, -\theta_-^\alpha, \quad (21)$$

so we are essentially dealing with a standard $\mathbf{R}^8/\mathbf{Z}_2$ supersymmetric orbifold.

This orbifold has well-known twist fields, which will receive contributions of the bosons and fermions. The bosonic twist operator σ is defined via the operator product

$$x_-^i(z) \cdot \sigma(0) \sim z^{-\frac{1}{2}} \tau^i(0) \quad (22)$$

Since the fermions transform in spinor representation $\mathbf{8}_s$, their spin fields $\Sigma^i, \Sigma^{\dot{\alpha}}$ will transform in the vector representation $\mathbf{8}_v$ and the conjugated spinor representation $\mathbf{8}_c$. They are related via the operator products

$$\begin{aligned} \theta_-^\alpha(z) \cdot \Sigma^i(0) &\sim z^{-\frac{1}{2}} \gamma_{\alpha\dot{\alpha}}^i \Sigma^{\dot{\alpha}}(0) \\ \theta_-^\alpha(z) \cdot \Sigma^{\dot{\alpha}}(0) &\sim z^{-\frac{1}{2}} \gamma_{\alpha\dot{\alpha}}^i \Sigma^i(0) \end{aligned} \quad (23)$$

The bosonic twist field σ and the spin fields Σ^i or $\Sigma^{\dot{\alpha}}$ have all conformal dimension $h = \frac{1}{2}$ (i.e. $\frac{1}{16}$ for each coordinate), and the conjugated field τ^i in (22) has dimension $h = 1$.

For the interaction vertex we propose the following space-time supersymmetric, $SO(8)$ invariant, weight $\frac{3}{2}$ field

$$\tau^i \Sigma^i \quad (24)$$

This twist field lies in the NS sector of the CFT, and represents an interaction between incoming and outgoing Ramond states.^{||} It can be written as the descendent of the chiral primary field $\sigma\Sigma^{\dot{\alpha}}$, since (no summation over $\dot{\alpha}$)

$$[G_{-\frac{1}{2}}^{\dot{\alpha}}, \sigma\Sigma^{\dot{\alpha}}] = \tau^i \Sigma^i. \quad (25)$$

This is a special case of the more general identity

$$[G_{-\frac{1}{2}}^{\dot{\alpha}}, \sigma\Sigma^{\dot{\beta}}] + [G_{-\frac{1}{2}}^{\dot{\beta}}, \sigma\Sigma^{\dot{\alpha}}] = \delta^{\dot{\alpha}\dot{\beta}} \tau^i \Sigma^i. \quad (26)$$

The interaction vertex operator satisfies

$$[G_{-\frac{1}{2}}^{\dot{\alpha}}, \tau^i \Sigma^i] = \partial_z(\sigma\Sigma^{\dot{\alpha}}), \quad (27)$$

as is clear by using the Jacobi identity and the above relation. For the full description we also have to include the right-moving degrees of freedom.

To obtain the complete form of the effective world-sheet interaction term we have to tensor the left-moving and right-moving twist fields and to sum over the pairs of I, J labeling the two possible eigenvalues that can be permuted by the \mathbf{Z}_2 twist

$$\sum_{I < J} \lambda \int d^2z \left(\tau^i \Sigma^i \otimes \bar{\tau}^j \bar{\Sigma}^j \right)_{IJ}. \quad (28)$$

This is a weight $(\frac{3}{2}, \frac{3}{2})$ conformal field. The corresponding coupling constant λ has therefore total dimension -1 and the interaction will scale linear in g_s just as needed, see eqn (18).

The interaction is space-time supersymmetric. First, it preserves the world-sheet $\mathcal{N} = 8$ supersymmetry representing the unbroken charges $Q^{\dot{\alpha}}$, since supersymmetric variations become total derivatives and we integrate over the string world-sheet. The broken space-time supersymmetries Q^{α} are trivially conserved. Although the fermion fields θ_-^{α} satisfy Ramond boundary conditions, and therefore pick up a minus sign in the local coordinate z when transported around $z = 0$, Q^{α} is proportional to the zero-mode of the linear sum θ_+^{α} , which is not broken by the twist field interaction.

It is interesting to compare the above twist field interaction with the conventional formalism of light-cone string theory^{**}. As is discussed by Mandelstam [18], in order to obtain a fully $SO(9, 1)$ Lorentz invariant interaction in the light-cone formalism, it does

^{||}Even though the fermion has Ramond spin structure, the supercurrent $G^{\dot{\alpha}}$ obeys NS periodicity. Using $N = 2$ language the chiral primary fields are $\sigma\Sigma^{\dot{i}}$ and $\sigma\Sigma^{\dot{\alpha}}$. After spectral flow they give Ramond ground states.

^{**}We thanks N. Berkovits for pointing out this relation to us.

not suffice to consider only the geometric joining and splitting interaction that in our formalism is represented by the insertion of the twist field $\sigma(0)$. The interaction has to be supplemented with a further operator insertion that reads in our notation

$$\oint \frac{dz}{z^{\frac{1}{2}}} \Sigma^i \partial x^i(z) \sigma(0) = \tau^i \Sigma^i(0). \quad (29)$$

In comparison with the Riemann surfaces picture of interacting strings [18] one should note that around the interaction point the usual string world-sheet is actually a double cover of the local coordinate z of our orbifold CFT. The above operator insertion should be tensored with a similar expression for the right-movers, giving precisely our interaction vertex (28). We further remark that the current $\Sigma^i \partial x^i$ has a simple interpretation as the $\mathcal{N} = 1$ world-sheet supercurrent in the covariant NSR formulation of the type II string. Indeed, in the NSR language the above operator insertion corresponds simply to picture changing. The fact that the twist field interactions (and the higher n -point vertices obtained through contact terms) respect the ten-dimensional Lorentz invariance is of course highly suggestive that the matrix string will also be Lorentz invariant.

HAMILTONIAN FORMULATION

It will be convenient in the following to represent the degrees of freedom in a Hamiltonian form, by introducing the standard conjugate variables (Π_i, X^i) for the scalar fields, and (E, A_1) , with E the 1-dimensional electric field, for the gauge fields. The Hamiltonian H and total 1-momentum P then take the form

$$H = \oint d\sigma T_{00} \quad P = \oint d\sigma T_{01} \quad (30)$$

where T_{00} and T_{01} are the components of the two-dimensional energy-momentum tensor, given by

$$\begin{aligned} T_{00} &= \frac{1}{2} \text{tr} \left(\Pi_i^2 + (DX_i)^2 + \theta^T \gamma^9 D\theta \right. \\ &\quad \left. + \frac{1}{g_s} \theta^T \gamma^i [X_i, \theta] + \frac{1}{g_s^2} (E^2 + [X^i, X^j]^2) \right) \\ T_{01} &= \text{tr} \left(\Pi_i DX^i + \theta^T D\theta \right) \end{aligned} \quad (31)$$

Here D is the YM covariant derivative along the spatial direction of the world-sheet and the γ -matrices are taken to be 16×16 matrices. The spacetime supercharges of the matrix string model take the form (now α denotes a non-chiral spinor and runs from 1 to 16)

$$\tilde{Q}^\alpha = \frac{1}{\sqrt{N}} \oint d\sigma \text{tr} \theta^\alpha, \quad Q^\alpha = \sqrt{N} \oint d\sigma G^\alpha, \quad (32)$$

with

$$G^\alpha = \text{tr} \left(\theta^T (\gamma^9 E + \gamma^{9i} D X_i + \gamma^i \Pi_i + \gamma^{ij} [X_i, X_j]) \right)^\alpha \quad (33)$$

COMPACTIFICATION

Following the current approach in matrix theory, the compactification of the matrix string on a torus T^d is achieved by reinterpreting the infinite dimensional matrices X^i as covariant derivatives D_i (written in a Fourier mode basis) of a large N gauge field defined in these extra dimensions [3, 7]. Adopting this procedure, the matrix string becomes equivalent to $d + 2$ -dimensional supersymmetric gauge field theory on the space-like manifold $S^1 \times T^d$. In this correspondence, the string coupling constant g_s is identified with square root of the volume of the torus T^d (in string units), and thus the free string limit amounts to shrinking the volume of the extra dimensions inside the T^d to zero. In general, like for the uncompactified theory, the large N limit of the gauge model needs to be accompanied by an appropriate IR-limit in the S^1 direction to find correspondence with the string theoretic degrees of freedom.

For the case $d = 2$, in which case the large N supersymmetric gauge theory lives in 3+1-dimensions, this exact rescaling was in fact considered previously in [19], see also [16].^{††} As also pointed out in [20, 21], the implied equivalence of large N four-dimensional SYM theory to type II string theory compactified on T^2 provides a natural explanation of the $SL(2, Z)$ S -duality symmetry of the former in terms of the T -duality of the latter. In the $g_s \rightarrow 0$ rescaling limit of the gauge theory, this connection between S -duality and T -duality of the resulting S_N -orbifold CFT was first pointed out in [16].

In general, the maximal number of dimensions one can compactify in this fashion is eight. In this case matrix theory becomes equivalent to $9 + 1$ dimensional large N SYM theory, whose IR behaviour should describe the type II string compactified to 1+1 dimensions. In principle one should be able to compactify one more dimension by taking the light-cone to be a cylinder. The finite N theory may seem to be a candidate for describing the sector of this compactification with given discrete momentum N along the extra S^1 . Although this procedure is adequate for computing the BPS spectrum of the theory, we suspect it gives an incomplete description of the dynamics of general non-BPS configurations.

FLUXES AND CHARGES

The above compactification procedure opens up the possibility of adding new charged objects to the theory, essentially by considering gauge theory configurations that carry non-trivial fluxes or other topological quantum numbers along the compactified directions.

^{††}The weak coupling limit $g_s \rightarrow 0$ was interpreted in [19] as a high energy limit, in which the typical length scale of excitations in two of the four directions is taken to be much smaller than in the other two transversal directions. It seems worthwhile to further elaborate this correspondence.

In particular, one can consider configurations with non-zero magnetic fluxes through the various two-cycles of the compactification torus T^d , and in this way one can introduce the D-membranes. More generally, using the correspondence with the matrix theory proposal of [3, 22], a (partially complete) list of fluxes and their type IIA interpretations are listed below

<u>2D SYM Theory</u>		<u>type IIA String</u>
$\oint \text{tr } E$	=	D-particle # q_0
$\oint \text{tr } \Pi_i$	=	momentum p_i
$\oint \text{tr } DX^i$	=	NS winding w_i
$\oint T_{01}$	=	NS winding w_+
$\oint \text{tr } [X^i, X^j]$	=	D-membrane # m_{ij}
$\oint T_{0i}$	=	D-membrane # m_{i+}
$\oint \text{tr } DX^{[i} X^j X^{k]}$	=	D-fourbrane # R_{ijk+}
$\oint \text{tr } X^{[i} X^j X^k X^{l]}$	=	NS fivebrane # W_{ijkl+}

Here T_{01} was defined in equation (31) and

$$T_{0i} = \text{tr} \left(EDX_i + \Pi^j [X_i, X_j] + \theta^T [X_i, \theta] \right)$$

which in the $d + 2$ dimensional SYM language is just the momentum flux in the i -th direction.

The above topological charges all appear as central terms in the supersymmetry algebra generated by the supercharges defined in (32). One finds [22]

$$\begin{aligned}
\{\tilde{Q}^\alpha, \tilde{Q}^\beta\} &= \delta^{\alpha\beta} \\
\{Q^\alpha, \tilde{Q}^\beta\} &= (\gamma^9 q_0 + \gamma^i p_i + \gamma^{9i} w_i + \gamma^{ij} m_{ij})^{\alpha\beta} \\
\{Q^\alpha, Q^\beta\} &= N(H + \gamma^9 w_+ + \gamma^i m_{i+} + \gamma^{9ijk} R_{ijk+} + \gamma^{ijkl} W_{ijkl+})^{\alpha\beta}
\end{aligned} \tag{34}$$

Here we recognize the central terms corresponding to the various charged (extended) objects that are present in the theory. In the last line, the complete right-hand side is proportional to N , and thus these terms diverge for $N \rightarrow \infty$ as soon as one of the central

charges m_{i+} , R_{ijk+} or W_{ijkl+} is non-zero. This corresponds to the fact that the 9th direction is necessarily decompactified in the large N limit, and thus these configurations represent string-like solitons with an infinite extent in the 9th direction.

D-PARTICLES

It is not immediately evident that the string, D-brane and fivebrane configurations as defined above will indeed behave exactly as in perturbative string theory. Here we will establish this correspondence for the case of the D-particle and the D-membrane. In particular, we will see that they indeed give rise to a new perturbative sector of strings, that satisfy Dirichlet boundary conditions on a corresponding codimension subspace.

First we consider a configuration with D-particle charge equal to q_0 . In the SYM language, this corresponds to a non-zero electric flux.

$$\frac{1}{2\pi} \oint d\sigma \operatorname{tr} E = q_0. \quad (35)$$

This correspondence can be understood in (at least) two ways. First, the electric flux arises upon compactification and T-duality as the KK-momentum in the extra 9th dimension. Since this direction was used to compactify M-theory to ten dimensions, this momentum gets interpreted as D-particle charge [1]. Alternatively, as indicated in *fig. 1*, after a T-duality and an S-duality, we can map our IIA strings to the D-strings of the type IIB theory. As shown in [5], fundamental type IIB strings attach to these D-strings by creating a non-zero electric flux on the SYM theory that described the D-string world-sheet dynamics. Inverting the above duality transformations, these fundamental IIB strings get identified with the D-particles in the IIA setup.

The simplest classical configuration that carries such an electric flux q_0 is

$$E = \mathbf{1}_{q_0 \times q_0}. \quad (36)$$

The presence of the electric flux will break the gauge group as

$$U(N) \rightarrow U(N - q_0) \times U(q_0). \quad (37)$$

The $U(N - q_0)$ sector, that does not carry an electric flux, represents the type IIA strings in the background of the D-particles. In the large N , $g_s \rightarrow 0$ limit this gives the usual free closed string spectrum. The $U(q_0)$ sector will describe the D-particle degrees of freedom and we will now examine it in more detail.

We have a similar breaking of the permutation group symmetry from S_N to $S_{N-q_0} \times S_{q_0}$. The symmetric group S_{q_0} describes the statistics of the D-particles. The Hilbert subspace that carries the electric flux q_0 will decompose (at least for weak string coupling) in twisted

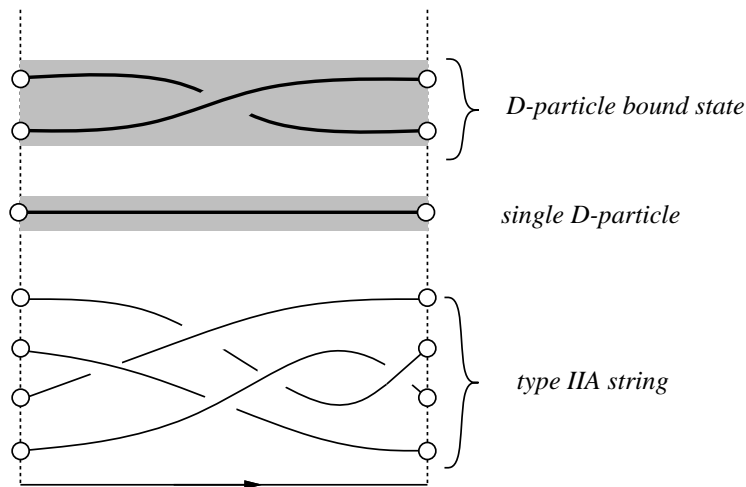


Fig. 4: If we put a flux in a finite rank subgroup of the SYM gauge group, the corresponding eigenvalues give “short strings” that represent the D-particles and their bound states. The “long strings” in the remaining sector describe the closed type IIA strings in the background of the D-particles.

sectors labeled by partitions of q_0 . These sectors have an interpretation as all the possible bound state configurations of the q_0 D-particles.

The eigenvalues of the $U(q_0)$ part of the matrices X^i can depend on the world-sheet parameter σ and thus a priori seem to describe strings. However, we would like to interpret them as D-particles. An important point is that in the large N limit we have to keep the total D-particle charge q_0 finite. This implies that the strings in the $U(q_0)$ sector become short strings with infinitely massive oscillations. These short string oscillations will therefore decouple at large N , leaving only their constant modes. These constant modes describe the positions of the D-particles and their various bound states. This behaviour should be contrasted with the eigenvalues in the remaining $U(N-q_0)$ sector, which can form the type IIA strings with the usual oscillation spectrum. Typical configurations thus consist of short strings describing the D-particles and long type IIA closed strings, as depicted in *fig. 4*.

It is worthwhile to note that here we naturally arrive at the existence of D-particle bound states, since these automatically arise as twisted sectors in the orbifold conformal field theory. So unlike for the original matrix model based on large N supersymmetric quantum mechanics, the existence of bound states is not an assumption but a direct consequence of well-established facts!

An electric flux of the form (36) will give a contribution $H = q_0/g_s^2$ to the SYM Hamiltonian. In the untwisted sector, that describes q_0 free D-particles, the usual mass-shell relation gives D-particle masses $m = 1/g_s$, which is the expected result. Similarly,

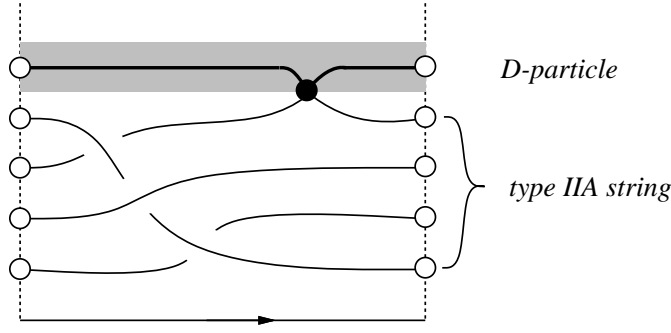


Fig. 5: In perturbation theory the closed string can get glued to the D-particle.

in a twisted sector that describes bound states of n_i D-particles, with $\sum n_i = q_0$, we find that the bound states have masses $m_i = n_i/g_s$. This is implied by the mass-shell relation, quite similarly as in our discussion of the free strings, since the contribution to the total L_0 is of the form

$$L_0^{tot} = \frac{q_0}{g_s^2} + \sum_i \frac{p_i^2}{n_i} \quad (38)$$

In particular, for a maximally twisted sector with one cycle of length q_0 we find $H = q_0/g_s^2 + p^2/q_0$ which implies $m = q_0/g_s$.

It should be emphasized that, as seen from eqn (38), the D-particles states just described in fact have very small momentum in the light-cone direction. Formally, they have $p_+ = \frac{n_i}{N} \rightarrow 0$ in the large N limit. It is clear, however, that one can give non-zero longitudinal momentum to the D-particles by attaching them (*i.e.* the short strings) to a long string.

Do these particles indeed interact with the type II strings as D-objects? In particular, we would like to see that the theory contains a sector of long strings, which attached to these particles in the usual fashion. The natural candidate for such configurations are twisted sectors in which (in spite of the symmetry breaking due to the presence of the electric flux) two different kind of eigenvalues are transposed, corresponding to respectively the long type IIA strings and the short string that represent the D-particles. In other words, the topology of the eigenvalues of the X^i fields in this sector is incompatible with topology of the eigenvalues of the (non-abelian) electric field E , in that the direction of the E field in the Lie algebra must necessarily point outside of the Cartan subalgebra specified by the X^i fields (and vice versa). In the IR limit this would however be energetically unfavorable, *unless* the short and long eigenvalue (that are transposed along the S^1) coincide. Where this happens the unbroken gauge group gets locally enhanced to $U(2)$, and this allows the eigenvalues to cross without much loss of energy, see *fig. 5*. In this way, the long string eigenvalue indeed stays glued to the short string eigenvalue representing the D-particle.

The ground state of such a bound state between a long and a short string represents the D-particle moving with non-zero p_+ . A quantitative verification of this picture is as follows. A long string of total length n occupies a $U(n)$ subgroup. If the string is in its ground state, all x_I essentially coincide and thus the $U(n)$ gauge symmetry is unbroken. If the configuration further carries an electric flux q_0 , this flux can lower its worldsheet energy by spreading into this unbroken gauge group^{‡‡}. The flux then takes the form of a flux $\frac{q_0}{n}$ inside the diagonal $U(1)$, together with an opposite 't Hooft type electric flux $\frac{q_0}{n}$ inside the $SU(n)$. The energy of this flux is now smaller by a factor of n , in exact accordance with the fact it now corresponds to a D-particle with finite $p_+ = n/N$.

These arguments are admittedly somewhat qualitative. It would be useful to perform a more quantitative study that supports the presented physical picture.

D-MEMBRANES

D-membranes are configurations with non-zero values for the topological charge

$$m_{ij} = \text{tr}[X_i, X_j]. \quad (39)$$

Let us briefly recall the construction of such configurations in the matrix model of M-theory and the resulting correspondence with the membrane world-volume theory. In $U(N)$ one can find two matrices U and V , such that

$$UV = e^{\frac{2\pi i}{N}} VU.$$

Any hermitian matrix X can then be written as

$$X = \sum_{nm} x_{nm} U^n V^m$$

and this expansion can be used to associate to X the function

$$x(p, q) = \sum_{n,m} x_{nm} e^{2\pi i(np+mq)}$$

The two coordinates (p, q) then become identified with the membrane surface. After implementing this transformation within the Hamiltonian of the large N SQM, one recovers the light-cone gauge membrane world-volume Hamiltonian [23, 24, 3]. In the matrix theory philosophy, this membrane configuration only needs to use a part of the total $U(N)$ gauge group. In particular, if we consider a sector of the theory with a membrane configuration in the 7-8 direction, we can decompose the total $U(N)$ matrix as

$$X^7 = p + x^7(p, q) + \tilde{X}^7$$

$$X^8 = q + x^8(p, q) + \tilde{X}^8$$

$$X^i = x^i(p, q) + \tilde{X}^i$$

^{‡‡}We thank T. Banks for suggesting this picture.

Here the first two terms describe the classical membrane and its transverse fluctuations, while \tilde{X}^i denotes the remaining part of X^i that commutes with the fluctuating membrane background.

We can now copy the same procedure in the matrix string theory. In this case the fields X becomes σ dependent, and thus at first sight the membrane coordinate fields $x(p, q)$ also acquire this additional dependence. However, just as for the D-particles, the presence of the membrane configuration breaks the total $U(N)$ gauge symmetry to a subgroup. In particular, this means the part of the eigenvalues that are occupied by the membrane can no longer be permuted. The p, q dependence does not allow such an action. So, in a similar fashion as in the case of the D-particle these eigenvalues necessarily correspond to short strings that can only be in their ground states. Therefore the functions $x^i(p, q)$ do not acquire a σ -dependency.

One should be able to analyze the interactions of the type IIA strings with the D-membrane quite explicitly along the lines outlined in the discussion of the D-particles. Again we expect that the “long” strings can now attach to “short” strings that constitute the membrane, analogous to the way depicted in *fig. 5*. However, in this case both end of the open string should be able to travel independently on the D-membrane world-volume. We leave a more precise analysis of these interactions for further study.

ACKNOWLEDGEMENTS

We would like to thank S. Bais, T. Banks, N. Berkovits, S. Das, F. Hacquebord, L. Motl, J-S. Park, W. Taylor, and B. de Wit for useful discussions. This research is partly supported by a Pionier Fellowship of NWO, a Fellowship of the Royal Dutch Academy of Sciences (K.N.A.W.), the Packard Foundation and the A.P. Sloan Foundation.

APPENDIX

In this appendix we derive the matrix string Hamiltonian starting from the matrix theory formalism of [3]. In particular we wish to identify the precise dependence on the string coupling constant. For a similar derivation see [11].

Starting point is the matrix theory Hamiltonian, written in eleven-dimensional Planckian units with $\ell_p = 1$ (we ignore numerical prefactors)

$$\mathcal{H} = R_{11} \operatorname{tr} \left(\Pi_i^2 + [X^i, X^j]^2 + \theta^T \gamma_i [X^i, \theta] \right). \quad (\text{A.1})$$

We now compactify the 9th dimension on a circle of radius R_9 . After the usual T-duality we can identify X^9 with the covariant derivative $R_9 D_\sigma$, where the coordinate σ runs from 0 to 2π . The conjugate momentum will be identified with the electric field E via $E = R_9 \Pi_9$. This gives the Hamiltonian (where $i = 1, \dots, 8$ now labels the transverse

coordinates)

$$\begin{aligned} \mathcal{H} = & \frac{R_{11}}{2\pi} \int \frac{d\sigma}{R_9} \text{tr} \left(\Pi_i^2 + R_9^2 (DX_i)^2 + R_9 \theta^T D\theta \right. \\ & \left. + \frac{1}{R_9^2} E^2 + [X^i, X^j]^2 + \theta^T \gamma_i [X^i, \theta] \right). \end{aligned} \quad (\text{A.2})$$

One can rescale the coordinates as $X^i \rightarrow R_9^{-1/2} X^i$ to find

$$\begin{aligned} \mathcal{H} = & \frac{R_{11}}{2\pi} \int d\sigma \text{tr} \left(\Pi_i^2 + (DX^i)^2 + \theta^T D\theta \right. \\ & \left. + \frac{1}{R_9^3} (E^2 + [X^i, X^j]^2) + \frac{1}{R_9^{3/2}} \theta^T \gamma_i [X^i, \theta] \right). \end{aligned} \quad (\text{A.3})$$

Conventionally, M-theory is related to type IIA string theory via the compactification of the 11th direction, which relates the string coupling constant g_s to $R_{11}^{3/2}$. To arrive at the matrix string point of view, however, we now interchange the role of the 9th and the 11th direction by defining the string scale $\ell_s = \sqrt{\alpha'}$ and string coupling constant g_s in terms of R_9 and the 11-dimensional Planck length ℓ_p

$$R_9 = g_s \ell_s, \quad \ell_p = g_s^{1/3} \ell_s, \quad (\text{A.4})$$

or equivalently $g_s = (R_9/\ell_p)^{3/2}$. From this we obtain the final result in string units $\ell_s = 1$

$$\begin{aligned} \mathcal{H} = & \frac{R_{11}}{2\pi} \int d\sigma \text{tr} \left(\Pi_i^2 + (DX^i)^2 + \theta^T D\theta \right. \\ & \left. + \frac{1}{g_s^2} (E^2 + [X^i, X^j]^2) + \frac{1}{g_s} \theta^T \gamma_i [X^i, \theta] \right). \end{aligned} \quad (\text{A.5})$$

In this convention, R_{11} normalizes the light-cone momentum p_+ via

$$p_+ = N/R_{11} \quad (\text{A.6})$$

and becomes infinite in the large N limit. The normalization chosen in the main text corresponds to total light-cone momentum $p_+ = 1$, so that $R_{11} = N$ and the mass-shell relation reads $p_- = H$. In addition we have absorbed this factor of N into the definition of the world-sheet time coordinate, so that the above Hamiltonian \mathcal{H} is related to the world-sheet time generator $H = L_0 + \bar{L}_0$ of the CFT as $\mathcal{H} = NH$.

References

- [1] E. Witten, “String Theory Dynamics in Various Dimensions,” Nucl. Phys. **B443** (1995) 85–126, [hep-th/9503124](#).

- [2] J. Schwarz, “The Power of M-theory,” *Phys. Lett.* **367B** (1996) 97, [hep-th/9510086](#).
- [3] T. Banks, W. Fischler, S. H. Shenker, and L. Susskind, “M Theory as a Matrix Model: A Conjecture,” [hep-th/9610043](#).
- [4] J. Polchinski, “Dirichlet-Branes and Ramond-Ramond Charges,” [hep-th/951001](#).
- [5] E. Witten, “Bound States of Strings and p -Branes,” *Nucl. Phys.* **B460** (1996) 335–350, [hep-th/9510135](#).
- [6] U.H. Danielsson, G. Ferretti, and B. Sundborg, “D-particle Dynamics and Bound States,” *Int. J. Mod. Phys.* **A11** (1996) 5463–5478, [hep-th/9603081](#);
D. Kabat and P. Pouliot, “A Comment on Zerobrane Quantum Mechanics,” *Phys. Rev. Lett.* **77** (1996) 1004–1007, [hep-th/9603127](#);
M. R. Douglas, D. Kabat, P. Pouliot, and S. H. Shenker, “D-branes and Short Distances in String Theory,” *Nucl. Phys.* **B485** (1997) 85–127, [hep-th/9608024](#).
- [7] W. Taylor, “D-brane Field Theory on Compact Spaces,” [hep-th/9611042](#).
- [8] R. Dijkgraaf, E. Verlinde and H. Verlinde, “BPS Spectrum of the Five-Brane and Black Hole Entropy,” *Nucl. Phys. B* **B486** (1997) 77–88, [hep-th/9603126](#).
- [9] R. Dijkgraaf, G. Moore, E. Verlinde, H. Verlinde, “Elliptic Genera of Symmetric Products and Second Quantized Strings,” *Commun. Math. Phys.* to be published, [hep-th/9608096](#).
- [10] T. Banks and N. Seiberg, “Strings from Matrices,” [hep-th/9702187](#).
- [11] L. Motl, “Proposals on Nonperturbative Superstring Interactions,” [hep-th/9701025](#).
- [12] T. Banks, lecture at Jerusalem Winter School, Jan 5, 1997.
- [13] S. Sethi and L. Susskind, “Rotational Invariance in the M(atrix) Formulation of Type IIB Theory,” [hep-th/9702101](#).
- [14] N. Ishibashi, H. Kawai, Y. Kitazawa, A. Tsuchiya, “A Large-N Reduced Model as Superstring,” [hep-th/9612115](#).
M. Li, “Strings from IIB Matrices,” [hep-th/9612222](#).
- [15] P.-M. Ho and Y.-S. Wu, “IIB/M Duality and Longitudinal Membranes,” [hep-th/9703016](#).
- [16] J.A. Harvey, G. Moore, and A. Strominger, “Reducing S -Duality to T -Duality,” *Phys.Rev.* **D52** (1995) 7161, [hep-th/9501022](#).
M. Bershadsky, A. Johansen, V. Sadov, C. Vafa, “Topological Reduction of 4D SYM to 2D σ -Model,” *Nucl. Phys.* **B448** (1995) 166, [hep-th/9501096](#).

- [17] M.B. Green, J.H. Schwarz, E. Witten, *Superstring Theory* (Cambridge University Press, 1987).
- [18] S. Mandelstam, “Lorentz Properties of the Three-String Vertex,” *Nucl. Phys.* **B83** (1974) 413–439; “Interacting-String Picture of the Fermionic String,” *Prog. Theor. Phys. Suppl.* **86** (1986) 163–170.
- [19] H. Verlinde and E. Verlinde, “QCD at High Energies and Two-Dimensional Field Theory,” [hep-th/9302104](#).
- [20] L. Susskind, “T Duality in M(atrix) Theory and S Duality in Field Theory,” [hep-th/9611164](#).
- [21] O. J. Ganor, S. Ramgoolam, and W. Taylor, “Branes, Fluxes and Duality in M(atrix)-Theory,” [hep-th/9611202](#).
- [22] T. Banks, N. Seiberg, and S. Shenker, “Branes from Matrices,” [hep-th/9612157](#).
- [23] B. de Wit, J. Hoppe, and H. Nicolai, “On the Quantum Mechanics of Supermembranes,” *Nucl. Phys.* **B305** (1988) 545.
D. Fairlie, P. Fletcher, and C. Zachos, *Phys. Lett.* **224B** (1989) 101–107.
- [24] P. K. Townsend, “The Eleven-Dimensional Supermembrane Revisited,” *Phys. Lett.* **B350** (1995) 184–187, [hep-th/9501068](#).